

# Productions of $\sigma$ meson in $J/\psi$ and $\psi(2s)$ decays

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## Abstract

Productions of scalar meson( $\sigma$ ) in the decays of  $J/\psi \rightarrow \omega + \sigma, J/\psi \rightarrow \gamma + \sigma$  and  $\psi(2s) \rightarrow \omega + \sigma, J/\psi + \sigma, \gamma + \sigma$  and  $ee^+$  annihilations in the regions of  $J/\psi$  and  $\psi$  are studied

It is known that  $\sigma$  meson( $f_0(600)$ ) is related to chiral symmetry which is one of the fundamental features of nonperturbative QCD. On the other hand, the identification of  $\sigma$  meson is a long standing puzzle[1]. Because of the large decay width( $\Gamma = (600 - 1000)MeV$ [1]) it is very difficult to establish the  $\sigma$  pole by a naive Breit-Wigner resonance[1].

In 1989 DM2 Collaboration[2] has observed a wide bump in the decays  $J/\psi \rightarrow \omega\pi^+\pi^-, \omega\pi^0\pi^0$  at a low ( $\pi\pi$ ) mass. A fit to a single Breit-Wigner curve folded to a  $\pi\pi$  phase space gives the following parameters

$$\begin{aligned}
M &= (414 \pm 20)MeV, \quad \Gamma = (494 \pm 58)MeV, \\
BR(J/\psi \rightarrow \omega(\pi^+\pi^-)_{lowmass}) &= (0.16 \pm 0.03) \times 10^{-2}, \\
BR(J/\psi \rightarrow \omega(\pi^0\pi^0)_{lowmass}) &= (0.08 \pm 0.01 \pm 0.02) \times 10^{-2}.
\end{aligned} \tag{1}$$

In 2004 BES Collaboration[3] has reported the measurements of a large broad peak due to  $\sigma$  meson in  $J/\psi \rightarrow \omega\pi^+\pi^-$  at low  $\pi\pi$  mass. Six analysis have been done and the results of mass and width are presented in Table 1 and 2 of Ref.[3].

Both DM2 and BES experiments of  $J/\psi \rightarrow \omega\pi\pi$  provide clear evidence for the existence of the scalar meson whose mass is low and width is wide. In Refs.[4] by parametrizing the amplitudes of the two pion decays of the  $\Upsilon(ns)$  and  $\psi(2s)$  the mass and width of  $\sigma(f_0(600))$  are determined.

The interesting points of the  $\sigma$  meson are 1)large decay width; 2)the mass and the decay

width are in the same range. Unlike other particles, the determination of the mass and the width of the  $\sigma$  meson is not an easy task[1] and it depends on model which is used to extract the parameters. The Table 2 of Ref.[3] clearly shows the model dependence of the mass and the width of the  $\sigma$  meson. When Breit-Wigner formula is used the mass parameter is often taken to be a constant and the width might depend on energy, see Ref.[3] for example. In Ref.[5] the pole approach is used to study the  $f_0(980)$ . Both real and imaginary parts of the pole are momentum dependent.

In this paper  $\sigma$  meson productions in the decays  $J/\psi \rightarrow \omega + \sigma, \gamma\sigma, ee^+ \rightarrow J/\psi \rightarrow \omega + \sigma$  and  $\psi(2s) \rightarrow \omega + \sigma, J/\psi + \sigma, \gamma + \sigma$  and  $ee^+ \rightarrow \psi(2s) \rightarrow \omega + \sigma, J/\psi + \sigma$  are studied by using effective Lagrangians. It is very interesting to notice that both  $J/\psi$  and  $\psi(2s)$  are very narrow resonances and  $\sigma$  is a very wide resonance.  $J/\psi$ ,  $\psi(2s)$ , and  $\omega$  are vector mesons. The Vector Meson Dominance(VMD) is used to predict the decay rate of  $J/\psi \rightarrow \gamma\sigma$  and to study  $\psi(2s) \rightarrow \gamma\sigma$ .

The  $\sigma$  meson has been observed in  $J/\psi \rightarrow \omega(2\pi)_{lowmass}$ [2,3], where the low mass region is  $2m_\pi$  to  $2m_K$ . Two decay modes,  $J/\psi \rightarrow \omega f_2(1270), b_1(1235)\pi$  have been measured[2], which contribute to  $J/\psi \rightarrow \omega + 2\pi$  in the region of higher mass of two pions. The tails of the two resonances contribute to the higher end of the mass distribution of  $(2\pi)_{lowmass}$  and the contributions are small. Estimation of the contribution of non-resonance to this process is needed. The larger branching ratio of  $J/\psi \rightarrow \rho\pi$  has been measured to be

$(1.27 \pm 0.09) \times 10^{-2}$ [6]. The coupling  $\pi\omega\rho$  of the Wess-Zumino-Witten anomaly[7] is expressed as

$$\mathcal{L}_{\pi\omega\rho} = -\frac{N_C}{\pi^2 g^2 f_\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \partial_\alpha \rho_\beta^i \pi^i, \quad (2)$$

where  $g$  is the universal coupling constant in the effective meson theory and is determined to be 0.39 by fitting  $\rho \rightarrow ee^+$ . It is similar to Eq.(2) the effective Lagrangian of the coupling  $J/\psi\rho\pi$  is constructed as

$$\mathcal{L}_{J/\psi\rho\pi} = g_\rho \epsilon^{\mu\nu\alpha\beta} \partial_\mu J_\nu \partial_\alpha \rho_\beta^i \pi^i, \quad (3)$$

where  $g_\rho$  is a parameter which is determined by the decay rate of  $J/\psi \rightarrow \rho\pi$

$$\Gamma(J/\psi \rightarrow \rho\pi) = \frac{3g_\rho^2}{8\pi} \left\{ \frac{1}{4m_J^2} (m_J^2 + m_\pi^2 - m_\rho^2)^2 - m_\pi^2 \right\}^{\frac{3}{2}} \quad (4)$$

to be  $g_\rho = 1.79 \times 10^{-3} GeV^{-1}$ . As a test of the effective Lagrangian(3), the decay rate of  $J/\psi \rightarrow \gamma\pi^0$  is calculated. Using the VMD[8]  $\rho^0 \rightarrow \frac{1}{2}egA_\mu$  in Eq.(3), we obtain

$$\Gamma(J/\psi \rightarrow \gamma\pi^0) = \frac{\alpha}{96} g_\rho^2 g^2 m_J^3 \left(1 - \frac{m_\pi^2}{m_J^2}\right)^3, \quad (5)$$

$$\Gamma(J/\psi \rightarrow \gamma\pi^0)/\Gamma(J/\psi \rightarrow \rho^0\pi^0) = 0.86 \times 10^{-2}. \quad (6)$$

The experimental value of this ratio(6) is  $0.93(1 \pm 0.45) \times 10^{-2}$ [6]. The ratio(6) is independent of  $g_\rho$ . Theory agrees with data very well. The effective Lagrangian(3) and the VMD work.

Using the two vertices(2,3), the contribution of this channel,  $J/\psi \rightarrow \rho\pi, \rho \rightarrow \omega\pi$  to

$J/\psi \rightarrow \omega\pi^+\pi^-$  is calculated

$$\Gamma(J/\psi \rightarrow \omega\pi^+\pi^-)_{\rho\pi,\rho \rightarrow \omega\pi} = 0.049keV, \quad (7)$$

The experimental value is  $\Gamma(J/\psi \rightarrow \omega\pi^+\pi^-) = 0.655(1 \pm 0.17)keV$ [6]. The contribution of  $J/\psi \rightarrow \rho\pi, \rho \rightarrow \omega\pi$  is about 7.5%. In the range of low mass of the two pions( $2m_\pi - 2m_K$ ) the contribution of this channel is about 7% of the the decay rate measured by DM2[2]. Therefore, in the low mass range the contributions of nonresonances are small and  $J/\psi \rightarrow \omega\sigma$  dominants the process of  $J/\psi \rightarrow \omega(\pi\pi)_{lowmass}$ .

The simplest effective Lagrangian of the coupling between  $\sigma$  field and pseudoscalars is constructed under SU(3) symmetry

$$\mathcal{L}_\sigma = g_\sigma \{ \pi^i \pi^i + K^+ K^- + K^0 \bar{K}^0 + \frac{1}{3} \eta \eta \} \sigma, \quad (8)$$

where the coupling constant  $g_\sigma$  is a parameter which can be determined by fitting the data of  $\sigma$  resonance. The Lagrangian(8) is used to fit the mass distribution of the two pions of the decay  $J/\psi \rightarrow \omega\pi^+\pi^-$ .

Using the Lagrangian(8), one-loop corrections of the propagator of the  $\sigma$  field are calculated. The imaginary part of the loop(pion-loop) is the decay width of  $\sigma \rightarrow \pi\pi$  and a mass correction is obtained from the real part. The couplings between  $\sigma$  meson and other particles, for example vector mesons and nucleons, contribute to the loop diagrams too. At low energies( $4m_\pi^2 < k^2 < 4m_K^2$ ,  $k$  is the momentum of  $\sigma$  meson) the loop diagrams from

these additional couplings don't have imaginary part and the real part are almost constants which can be absorbed into the bare mass of the  $\sigma$  meson(see below).

Taking one loop diagrams of pions, kaons, and  $\eta$  into account, the propagator of the  $\sigma$  field is written as

$$\Delta(k^2) = \frac{1}{k^2 - m_0^2 + \Pi(k^2)} = \frac{1}{k^2 - m^2(k^2) + i\sqrt{k^2}\Gamma_\sigma(k^2)}, \quad (9)$$

where  $m_0^2$  is the sum of the bare mass of the  $\sigma$  meson and the constants separated from the loop diagram, in the energy region  $4m_\pi^2 < k^2 < 4m_K^2$   $Im\Pi(k^2)$  is obtained only from the pion loop

$$Im\Pi(k^2) = \sqrt{k^2}\Gamma_\sigma(k^2),$$

$$\Gamma_\sigma(k^2) = \frac{3g_\sigma^2}{8\pi} \frac{1}{\sqrt{k^2}} \left(1 - \frac{4m_\pi^2}{k^2}\right)^{\frac{1}{2}}, \quad (10)$$

$$Re\Pi(k^2)_\pi = \frac{3g_\sigma^2}{8\pi^2} z_\pi \log \frac{1+z_\pi}{1-z_\pi} = \frac{g_\sigma^2}{8\pi^2} f_\pi(k^2),$$

$$Re\Pi(k^2)_K = \frac{g_\sigma^2}{2\pi^2} z_K \operatorname{atan} \frac{1}{z_K} = \frac{g_\sigma^2}{4\pi^2} f_K(k^2)$$

$$Re\Pi(k^2)_\eta = \frac{g_\sigma^2}{4\pi^2} \frac{1}{9} z_\eta \operatorname{atan} \frac{1}{z_\eta} = \frac{g_\sigma^2}{4\pi^2} f_\eta(k^2),$$

$$z_\pi = \left(1 - \frac{4m_\pi^2}{k^2}\right)^{\frac{1}{2}}, \quad z_K = \left(\frac{4m_K^2}{k^2} - 1\right)^{\frac{1}{2}}, \quad z_\eta = \left(\frac{4m_\eta^2}{k^2} - 1\right)^{\frac{1}{2}}, \quad (11)$$

$$m^2(k^2) = m_0^2 + Re\Pi(k^2). \quad (12)$$

The numerical results of the three functions,  $f_\pi(k^2)$ ,  $f_K(k^2)$ ,  $f_\eta(k^2)$  are shown in Fig.1. In the energy region the real part of the pion loop increases with energy fast; the kaon loop

decreases with energy slowly; the contribution of  $\eta$  loop is small and it doesn't vary with  $k^2$ . The calculation shows that if the mass of the particle is at the value of  $m_\rho$  or above the function is almost a constant in the energy region. Therefore, in this energy region the effects of heavier particle loops can be absorbed into the mass parameter  $m_0^2$ .

Usually, in a complete field theory the divergent part of this kind of loop diagram is used to renormalize the mass and the field. As mentioned above the divergent term of the loop diagram has been absorbed by the mass parameter  $m_0$ . The renormalization constant of the  $\sigma$  field can be defined too and it can be used to redefine the coupling constant  $g_\sigma$  which is determined by fitting data. This procedure doesn't affect the shape of the distribution of Eq(9) as a function  $k^2$  and Eq.(9) is used to fit data. There are two parameters:  $m_0$  and  $g_\sigma$ .

In the region of  $J/\psi$  resonance besides the coupling(8)  $J/\psi\omega\sigma$ ,  $\gamma J/\psi$ , and  $\gamma\omega\sigma$  couplings contribute to  $J/\psi \rightarrow \omega\sigma, \sigma \rightarrow 2\pi$  and  $ee^+ \rightarrow \omega\sigma, \sigma \rightarrow 2\pi$ . It is well known that at low energies in the VMD  $\rho, \omega, \phi$  mesons play dominant roles. In Ref.[9] the VMD has been extended to  $J/\psi$  meson to study  $\eta_c \rightarrow \gamma\gamma$ ,  $\psi(3872) \rightarrow \gamma\sigma$  and  $ee^+ \rightarrow \psi(3872) + \sigma$

$$\mathcal{L}_{\gamma J} = eg_J \left\{ -\frac{1}{2} F_{\mu\nu} (\partial_\mu J_\nu - \partial_\nu J_\mu) + A_\mu j^\mu \right\}, \quad (13)$$

The Lagrangian of  $J\omega\sigma$  coupling is constructed as

$$\mathcal{L}_{J\omega\sigma} = g_1 (\partial_\mu J_\nu - \partial_\nu J_\mu) (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) \sigma. \quad (14)$$

Eq.(13) is the standard form of the VMD of  $J/\psi$ , the parameter  $g_J$  is determined by fitting

the decay rate of  $J/\psi \rightarrow ee^+$ ,  $g_J = 0.0917$ . The decay rate of  $J/\psi \rightarrow \omega\sigma$  determines the coupling constant  $g_1$ ,  $j_\mu$  of Eq.(13) is obtained from Eq.(14) by replacing  $J/\psi$  by a photon field

$$j_\mu \rightarrow eg_J A_\mu. \quad (15)$$

Using this substitution(15), the Lagrangian of the coupling  $\gamma\omega\sigma$  is obtained from the Eq.(14)

$$\mathcal{L}_{\gamma\omega\sigma} = eg_J g_1 (\partial_\mu A_\nu - \partial_\nu A_\mu) (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) \sigma. \quad (16)$$

On the other hand, the Lagrangian of  $J\gamma\sigma$  is obtained by using the VDM substitution

$$\omega \rightarrow \frac{1}{6} eg A, \quad (17)$$

$$\mathcal{L}_{J\gamma\sigma} = \frac{1}{6} eg g_1 (\partial_\mu J_\nu - \partial_\nu J_\mu) (\partial_\mu A_\nu - \partial_\nu A_\mu) \sigma. \quad (18)$$

The expressions of Eqs.(16,18) show that current conservation is satisfied in both Lagrangians and Eqs.(16,18) are used to study physical processes in this paper. Current conservation is the reason why the effective Lagrangian  $\mathcal{L}_{J\omega\sigma}$  is constructed as Eq.(14). Using Eqs.(14,8) it is obtained

$$\begin{aligned} \frac{d\Gamma}{dk^2}(J/\psi \rightarrow \omega\pi^+\pi^-) &= \frac{g_1^2}{18\pi^2} \frac{1}{m_J^3} \{(m_J^2 - m_\omega^2 - k^2)^2 - 4m_\omega^2 k^2\}^{\frac{1}{2}} \\ &\quad \{m_J^2 m_\omega^2 + \frac{1}{2}(m_J^2 + m_\omega^2 - k^2)^2\} \frac{\sqrt{k^2} \Gamma_\sigma(k^2)}{(k^2 - m^2(k^2))^2 + k^2 \Gamma_\sigma(k^2)}, \end{aligned} \quad (19)$$



where  $k^2$  is the invariant mass of the two pions. The distribution,  $\frac{1}{g_1^2} \frac{d\Gamma}{d\sqrt{k^2}}(J/\psi \rightarrow \omega\pi^+\pi^-)$ , is shown in Fig.2. The two parameters,  $m_0$  and  $g_\sigma$  are chosen as

$$m_0 = 0.35GeV, \quad g_\sigma = 1.31GeV,$$

to fit the width and mass of the  $\sigma$  meson, which are determined to be

$$m_\sigma = 0.579GeV, \quad \Gamma_\sigma = 0.460GeV.$$

The values of  $m_\sigma$  and  $\Gamma_\sigma$  are compatible with the DM2 and BES data.  $\Gamma_\sigma = 0.309GeV$  is obtained from the direct calculation of  $\Gamma_\sigma(10)$ . It is different from the width at half of the height of the  $\sigma$  resonance. This is the effect of wide resonance.

Integrating over  $k^2$ , inputting the decay rate of  $J/\psi \rightarrow \omega\sigma$ [2], and subtracting the background  $J/\psi \rightarrow \rho\pi, \rho \rightarrow \omega\pi$ , it is determined

$$g_1^2 = 0.61(1 \pm 0.22)10^{-6}GeV^{-2}.$$

Now the  $\sigma(ee^+ \rightarrow J/\psi \rightarrow \omega\sigma, \sigma \rightarrow \pi^+\pi^-)$  can be calculated.  $J/\psi$  is a very narrow resonance. Before the calculation of this cross section we use  $ee^+ \rightarrow \mu\mu^+$  in the region of  $J/\psi$  resonance to study the effect of narrow resonance. In the literature(see Ref.[10] for example) this process is described by one-photon exchange and  $J/\psi$  resonance ( $\gamma, \gamma - J/\psi - \gamma$ ) and using Eq.(13) the cross section is expressed as

$$\frac{d\sigma}{d\cos\theta}(ee^+ \rightarrow \mu\mu^+) = \frac{\pi\alpha^2}{2} \frac{1}{q^2} \left| 1 + \frac{e^2 g_J^2 q^2}{q^2 - m_J^2 + i\sqrt{q^2}\Gamma_J} \right|^2 (1 + \cos^2\theta). \quad (20)$$

Because  $J/\psi$  is a very narrow resonance at the peak,  $q^2 = m_J^2$ , the value of the resonance term is very large  $\frac{e^2 g_J^2 q^2}{i\sqrt{q^2}\Gamma_J} = -25.4i \equiv x$ . In field theory besides the diagrams mentioned above there are chain diagrams,  $\gamma, \gamma - J/\psi - \gamma, \gamma - J/\psi - \gamma - J/\psi - \gamma, \dots$ . Taking one photon propagator out a series  $1 + x + x^2 + \dots$  is obtained. The value of  $x$  is large and perturbation at the peak of the resonance doesn't work. Therefore, all the chain diagrams of photon and  $J/\psi$  resonance should be taken into account and  $1 + x + x^2 + \dots = \frac{1}{1-x}$  is obtained. In the region of the  $J/\psi$  resonance after the chain diagrams of  $J/\psi$  resonance are added up the photon propagator is modified to be

$$\frac{1}{q^2} \left\{ 1 - \frac{e^2 g_J^2 q^2}{q^2 - m_J^2 + i\sqrt{q^2}\Gamma_J} \right\}^{-1}. \quad (21)$$

Using Eq.(21), the cross section of  $ee^+ \rightarrow \mu\mu^+$  in the region of  $J/\psi$  resonance is rewritten as

$$\frac{d\sigma}{d\cos\theta}(ee^+ \rightarrow \mu\mu^+) = \frac{\pi\alpha^2}{2} \frac{1}{q^2} \left| 1 - \frac{e^2 g_J^2 q^2}{q^2 - m_J^2 + i\sqrt{q^2}\Gamma_J} \right|^{-2} (1 + \cos^2\theta). \quad (22)$$

The comparison between theory(22) and the data[11] is shown in Fig.3. Theory agrees with data well.

Using the effective Lagrangians(13,14,18) and the modified photon propagator(21), the cross section of  $ee^+ \rightarrow \omega\sigma, \sigma \rightarrow \pi^+\pi^-$  in the  $J/\psi$  region is obtained

$$\frac{d\sigma}{dk^2} = \frac{12\pi}{q^4} \frac{1}{\left| 1 - \frac{e^2 g_J^2 q^2}{q^2 - m_J^2 + i\sqrt{q^2}\Gamma_J} \right|^2} \frac{m_J^4 + q^2\Gamma_J^2}{(q^2 - m_J^2)^2 + q^2\Gamma_J^2} \Gamma(J/\psi \rightarrow ee^+) \frac{d\Gamma}{dk^2}(J/\psi \rightarrow \omega\sigma, \sigma \rightarrow \pi^+\pi^-), \quad (23)$$

where  $\Gamma(J/\psi \rightarrow ee^+) = \frac{4}{3}\pi\alpha^2 g_J^2 \sqrt{q^2}$  and replacing  $m_J$  by  $\sqrt{q^2}$  in Eq.(19),  $\frac{d\Gamma}{dk^2}$  is obtained. Integrating over  $k^2$  from  $4m_\pi^2$  to  $4m_K^2$ , the cross section in the low mass region of  $\pi\pi$  is obtained and shown in Fig.4.

The decay rate of  $J/\psi \rightarrow \gamma\sigma, \sigma \rightarrow \pi^+\pi^-$  is predicted by Eq.(18)

$$\begin{aligned}\Gamma(J/\psi \rightarrow \gamma(\pi^+\pi^-)_{lowmass}) &= \frac{\alpha g^2 g_1^2 g_\sigma^2}{216(2\pi)^2 m_J^3} \int_{4m_\pi^2}^{4m_K^2} dk^2 \frac{(m_J^2 - k^2)^3}{(q^2 - m^2(k^2))^2 + k^2 \Gamma_\sigma^2(k^2)} \left(1 - \frac{4m_\pi^2}{k^2}\right)^{\frac{1}{2}} \\ B(J/\psi \rightarrow \gamma(\pi^+\pi^-)_{lowmass}) &= 0.19 \times 10^{-7}.\end{aligned}\quad (24)$$

The branching ratio is very small. This decay channel,  $J/\psi \rightarrow \gamma\sigma$ , has been measured and not been found by BES[3]. The theory is consistent with BES's search.

The productions of  $\sigma$  meson in  $\psi(2s)$  decays can be studied by the same theoretical approach.

$\psi(2s)$  is a narrow resonance too. In the region of  $\psi(2s)$  the cross section of  $ee^+ \rightarrow \mu\mu^+$  is expressed as

$$\sigma = \frac{4\pi\alpha^2}{3} \frac{1}{q^2} \left| 1 - \frac{e^2 g_\psi^2 q^2}{q^2 - m_\psi^2 + i\sqrt{q^2} \Gamma_\psi} \right|^{-2}, \quad (25)$$

where  $g_\psi$  is the coupling constant between photon and  $\psi(2s)$  and is determined to be  $g_\psi = 0.051$  by fitting the decay rate of  $\psi(2s) \rightarrow ee^+$ . The comparison with data[12] is shown in Fig.5.

$\psi(2s) \rightarrow \omega\pi^+\pi^-$  has been measured[13]. Like  $J/\psi \rightarrow \omega\pi^+\pi^-$ [2,3], the channels  $b_1\pi, \omega f_2(1270)$  dominate  $\psi(2s) \rightarrow \omega\pi^+\pi^-$ . There is small  $B(\psi(2s) \rightarrow \omega(\pi^+\pi^-)_{lowmass})$ , where the invariant

mass of two pions is in the range  $2m_\pi < m_{2\pi} < 2m_K$ . Based on the 12% rule of the decays of  $\psi(2s)$  and  $J/\psi$  and using DM2 data[2], it is estimated

$$B(\psi(2s) \rightarrow \omega(\pi^+\pi^-)_{lowmass}) = (1.92 \pm 0.36) \times 10^{-4}. \quad (26)$$

This value is consistent with BES data[13] within the large experimental errors. The isospin of  $\pi^+\pi^-$  is zero and the branching ratio of  $\psi(2s) \rightarrow \omega(\pi^0\pi^0)_{lowmass}$  should be half of the value(26).

Comparing with  $J/\psi \rightarrow \omega(\pi\pi)_{lowmass}$ , it is expected that  $\psi(2s) \rightarrow \omega\sigma, \sigma \rightarrow 2\pi$  is the main source of  $\psi(2s) \rightarrow \omega(\pi^+\pi^-)_{lowmass}$ . In Eq.(19) replacing  $m_J$  and  $g_1$  by  $m_\psi$  and  $g_1(2s)$  respectively, the distribution  $\frac{1}{g_1^2(2s)} \frac{d\Gamma}{dk^2}(\psi(2s) \rightarrow \omega\sigma, \sigma \rightarrow \pi^+\pi^-)$  is shown in Fig.6.  $g_1(2s)$  is determined by the branching ratio (26) to be  $g_1(2s) = 0.37 \times 10^{-3} GeV^{-1}$ . It is similar to Eq.(23), in  $\psi(2s)$  region  $\sigma(ee^+ \rightarrow \omega\pi^+\pi^-)$  is obtained

$$\frac{d\sigma}{dk^2} = \frac{12\pi}{q^4} \frac{1}{|1 - \frac{e^2 g_\psi^2 q^2}{q^2 - m_\psi^2 + i\sqrt{q^2}\Gamma_\psi}|^2} \frac{m_\psi^4 + q^2\Gamma_\psi^2}{(q^2 - m_\psi^2)^2 + q^2\Gamma_\psi^2} \Gamma(\psi \rightarrow ee^+) \frac{d\Gamma}{dk^2}(\psi \rightarrow \omega\sigma, \sigma \rightarrow \pi^+\pi^-), \quad (27)$$

The results are shown in Fig.7.  $\sigma(ee^+ \rightarrow \omega\pi^+\pi^-)$  in  $\psi(2s)$  region is much smaller than  $\sigma(ee^+ \rightarrow \omega\pi^+\pi^-)$  in  $J/\psi$  region.

The decays  $\psi(2s) \rightarrow J/\psi\pi^+\pi^-, J/\psi\pi^0\pi^0$  have been measured [12,14]. The ratio of the two decay rates is about 2. The isospin of two pions is zero. The distribution of the invariant mass of two pion shows a bump in the kinematic region. There are many theoretical studies

on the  $\pi\pi$  decay channels[15]. It is interesting to notice that the bump of the invariant mass of two pions is just the region of the mass of  $\sigma$  meson. Therefore, it is possible that  $\sigma$  meson is produced in the decay  $\psi(2s) \rightarrow J/\psi 2\pi$ [4]. In this paper effective Lagrangian has been used to study the  $\sigma$  productions in  $J/\psi, \psi(2s) \rightarrow \omega\pi\pi$ . The mass and the width of  $\sigma$  meson determined are consistent with experimental fits. The prediction of  $J/\psi \rightarrow \gamma\sigma$  agrees with BES's search. Now the same method can be used to study  $\psi(2s) \rightarrow J/\psi + \sigma, \sigma \rightarrow \pi\pi$ .

It is similar to Eqs.(13,14) the effective Lagrangians related to  $\psi(2s)$  are constructed as

$$\mathcal{L}_{\gamma\psi} = eg_{\psi}\{-\frac{1}{2}F_{\mu\nu}(\partial_{\mu}\psi_{\nu} - \partial_{\nu}\psi_{\mu}) + A_{\mu}j^{\mu}\}, \quad (28)$$

$$\mathcal{L}_{\psi J\sigma} = g_2(\partial_{\mu}\psi_{\nu} - \partial_{\nu}\psi_{\mu})(\partial_{\mu}J_{\nu} - \partial_{\nu}J_{\mu})\sigma, \quad (29)$$

where  $j_{\mu}$  is obtained from Eq.(29) by the substitution  $\psi(2s) \rightarrow eg_{\psi}A_{\mu}$ . Replacing  $g_1, m_J$ , and  $m_{\omega}$  by  $g_2, m_{\psi}$ , and  $m_J$  in Eq.(19) respectively, the distribution  $\frac{1}{g_2^2} \frac{d\Gamma}{dk^2}(\psi \rightarrow J/\psi \pi^+ \pi^-)$  is obtained and is plotted in Fig.8. which shows that the distribution  $\frac{1}{g_2^2} \frac{d\Gamma}{d\sqrt{k^2}}$  is pretty flat in the range of  $\sqrt{k^2} < 0.5 GeV$ . This behavior is inconsistent with BES's data(see Fig.6 of Ref.[14]). In this theoretical approach if we adjust the decay width of  $\sigma$  to a lower value(about 200 MeV) (reduce the parameter  $g$  from 0.065 to 0.03) the distribution showed in Fig.6 of Ref.[14] can be fitted very well. However, the narrow  $\sigma$ -resonance cannot fit the data pf  $J/\psi \rightarrow \omega(\pi\pi)_{lowmass}$  and inconsistent with all other findings(wide  $\sigma$  resonance). Therefore, from this theoretical approach we can only say that the decays  $\psi(2s) \rightarrow J/\psi + \sigma$

might be part of  $\psi(2s) \rightarrow J/\psi\pi\pi$  and it might not be the main source of  $\psi(2s) \rightarrow J/\psi + \pi\pi$ .

We cannot predict the branch ratio of  $\psi(2s) \rightarrow J/\psi + \sigma$ .

The phase space of  $\psi \rightarrow J/\psi\pi^+\pi^-$  is limited. Measurements of  $\psi(2s) \rightarrow \gamma(\pi\pi)_{lowmass}$  can provide useful information about the decay channel  $\psi(2s) \rightarrow J/\psi + \sigma$ . The decay rate of  $\psi \rightarrow \gamma\pi^+\pi^-$  has been measured[16] in the range of  $m_{\pi\pi} > 0.9GeV$ . The distribution of the invariant mass of two pions of  $\psi \rightarrow \gamma + \sigma, \sigma \rightarrow \pi^+\pi^-$  can be predicted by this approach of effective Lagrangian. Using the VMD of  $J/\psi$ [9], it is obtained from Eq.(29)

$$\mathcal{L}_{\psi\gamma\sigma} = eg_2g_J(\partial_\mu\psi_\nu - \partial_\nu\psi_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)\sigma. \quad (30)$$

There is another term which is obtained from  $\mathcal{L}_{\psi\omega\sigma}$  by the substitution  $\omega \rightarrow \frac{1}{6}gA$ . Using the values of  $g$  and  $g(2s)$ , the contribution of this new term is negligible. From Eqs.(30,8) it is obtained

$$\Gamma(\psi \rightarrow \gamma(\pi^+\pi^-)_{lowmass}) = \frac{\alpha g_J^2 g_2^2 g_\sigma^2}{6(2\pi)^2 m_J^3} \int_{4m_\pi^2}^{4m_K^2} dk^2 \frac{(m_\psi^2 - k^2)^3}{(q^2 - m^2(k^2))^2 + k^2 \Gamma_\sigma^2(k^2)} \left(1 - \frac{4m_\pi^2}{k^2}\right)^{\frac{1}{2}} \quad (31)$$

The distribution  $\frac{1}{g_2^2} \frac{d\Gamma}{d\sqrt{k^2}}$  of  $\psi(2s) \rightarrow \gamma\sigma, \sigma \rightarrow (2\pi)_{lowmass}$  is predicted(Fig.9), where  $2m_\pi < m(2\pi) < 2m_K$ . The mass and the width determined from this process are the same as the ones determined from  $J/\psi \rightarrow \omega(\pi\pi)_{lowmass}$ . We cannot predict the decay rate.

In summary, the effective Lagrangians and the VMD are used to study the productions of the  $\sigma$  meson in the decays of  $J/\psi$  and  $\psi(2s)$  and  $ee^+$  annihilations. The decay rates

of  $J/\psi \rightarrow \gamma\sigma$  and  $\psi(2s) \rightarrow \omega\sigma$  are predicted.  $\psi(2s) \rightarrow J/\psi\sigma$  might not be the dominant process for  $\psi(2s) \rightarrow J/\psi\pi\pi$ .

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## Figure Captions

**Fig. 1** Real parts of the pion(top), kaon, and  $\eta$  loops(bottom).

**Fig. 2** Distribution  $\frac{d\Gamma}{d\sqrt{k^2}}(J/\psi \rightarrow \omega\pi^+\pi^-)$ .

**Fig.3**  $\sigma(ee^+ \rightarrow \mu\mu^+)$  in the region of  $J/\psi$ ,  $\cos\theta < 0.6$ .

**Fig.4**  $\sigma(ee^+ \rightarrow \omega + \pi^+\pi^-)$  in the region of  $J/\psi$ ,  $4m_\pi^2 < k^2 < 4m_K^2$ .

**Fig.5**  $\sigma(ee^+ \rightarrow \mu\mu^+)$  in the region of  $\psi(2s)$ .

**Fig.6** Distribution  $\frac{1}{g_1^2(2s)} \frac{d\Gamma}{d\sqrt{k^2}}(\psi(2s) \rightarrow \omega\pi^+\pi^-)$ .

**Fig.7**  $\sigma(ee^+ \rightarrow \omega + \pi^+\pi^-)$  in the region of  $\psi(2s)$ ,  $4m_\pi^2 < k^2 < 4m_K^2$ .

**Fig.8** Distribution  $\frac{1}{g_2^2} \frac{d\Gamma}{d\sqrt{k^2}}(\psi(2s) \rightarrow J/\psi + \pi^+\pi^-)$ .

**Fig.9** Distribution  $\frac{1}{g_2^2} \frac{d\Gamma}{d\sqrt{k^2}}(\psi(2s) \rightarrow \gamma\sigma, \sigma \rightarrow \pi^+\pi^-)$ .

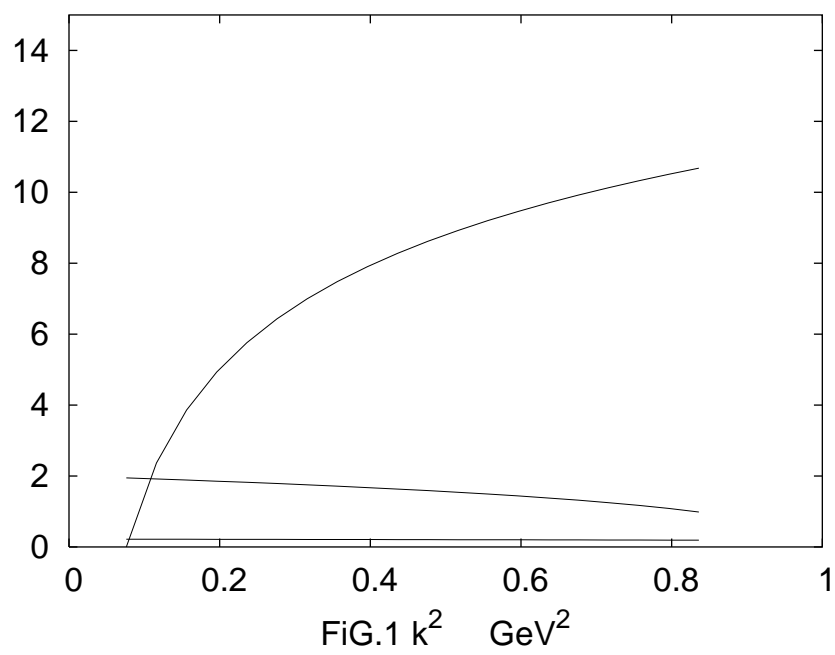


FIG. 1.

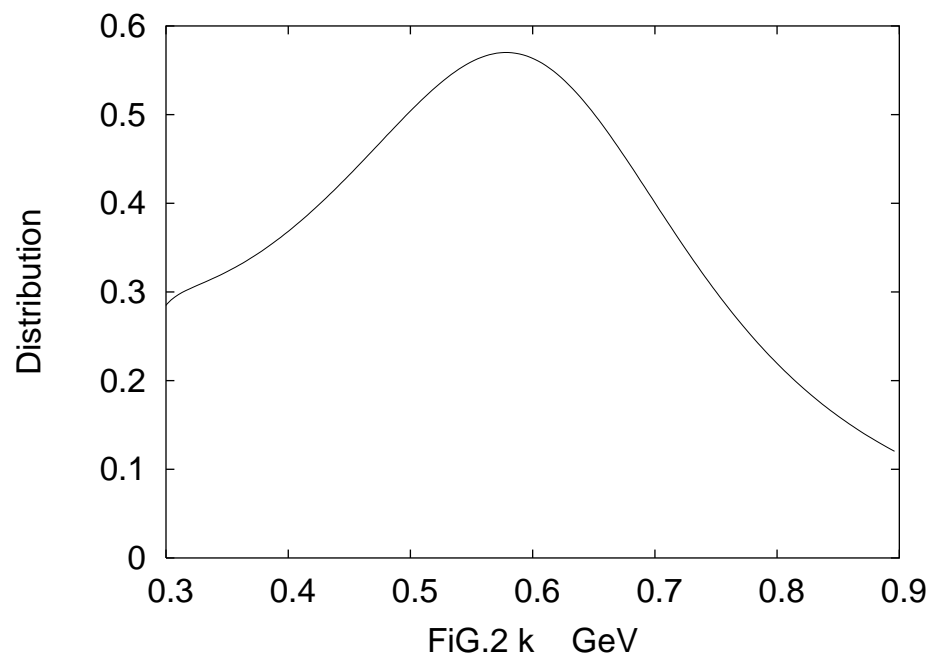


FIG. 2.

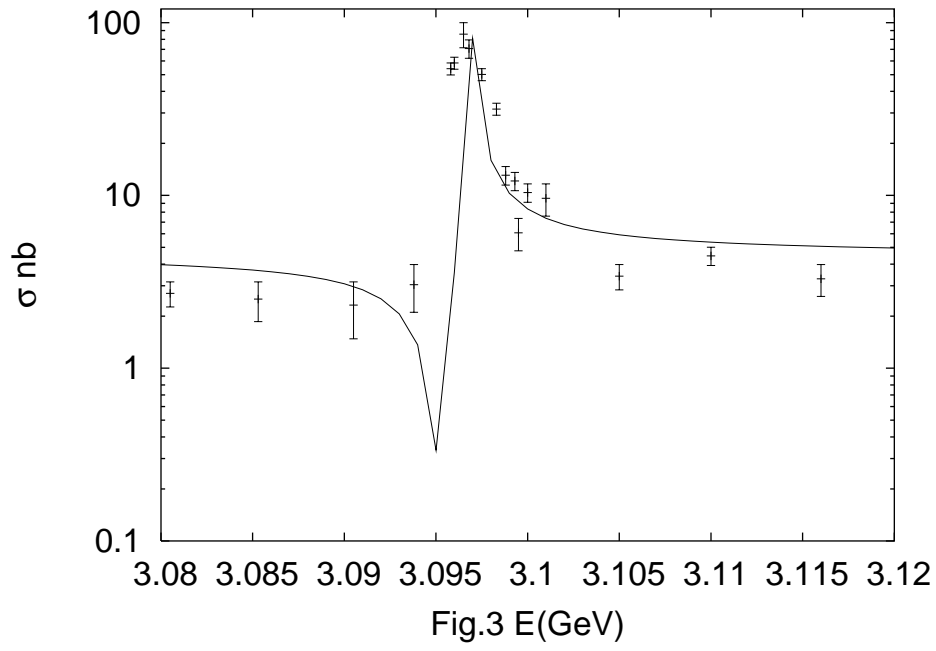


FIG. 3.

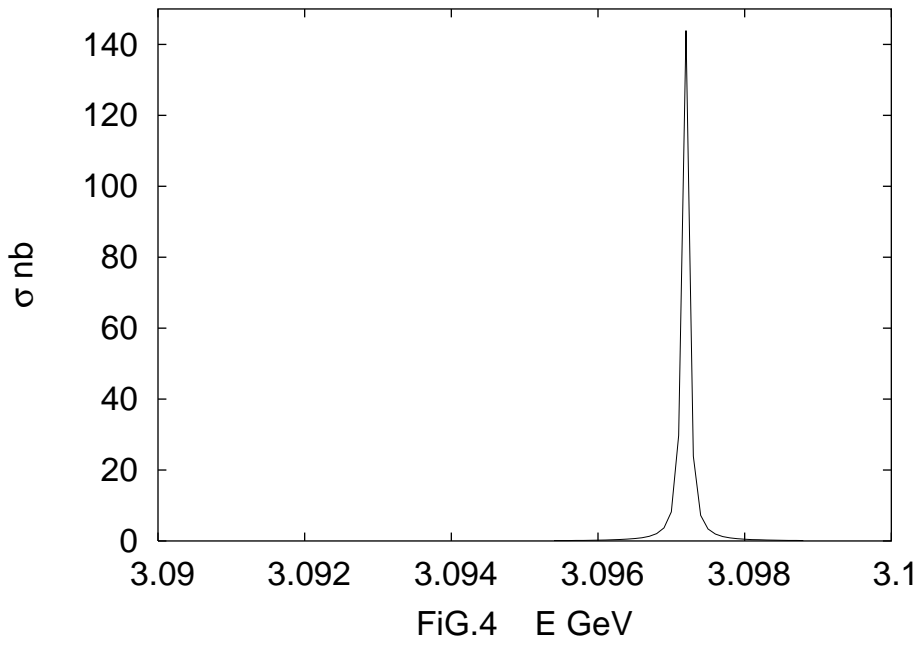


FIG. 4.

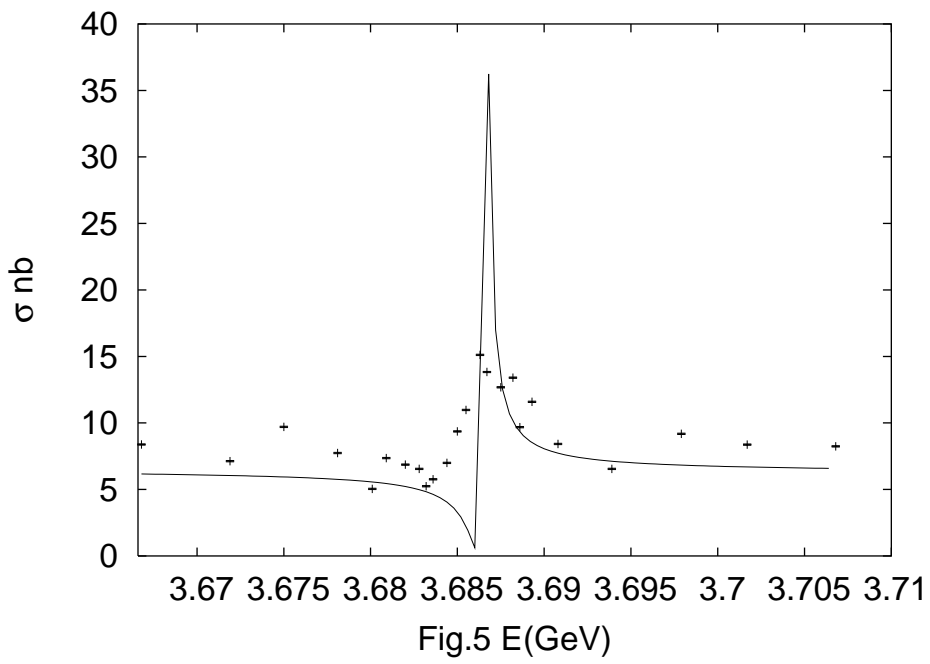


FIG. 5.

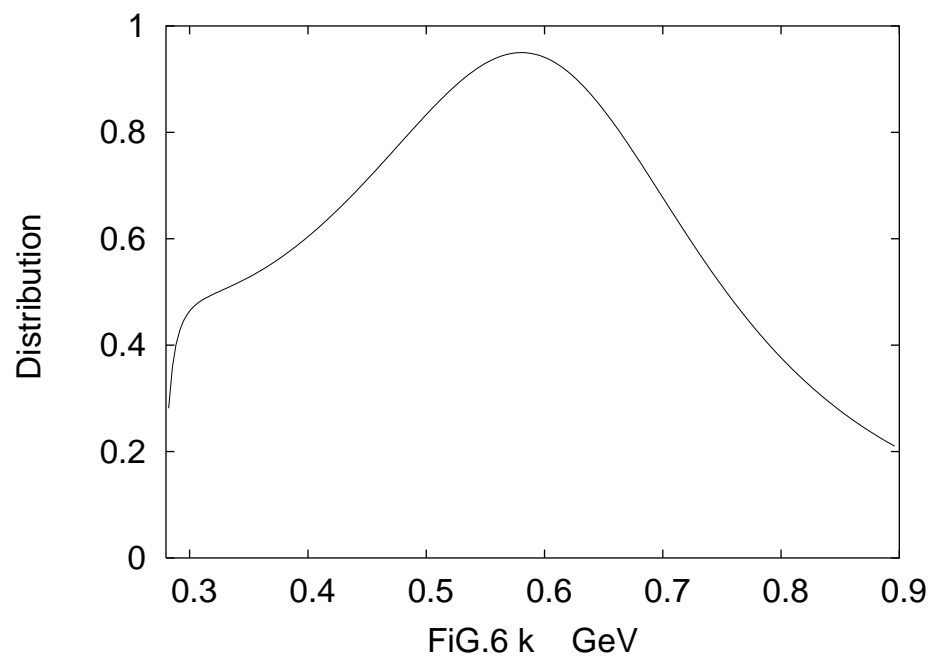


FIG. 6.

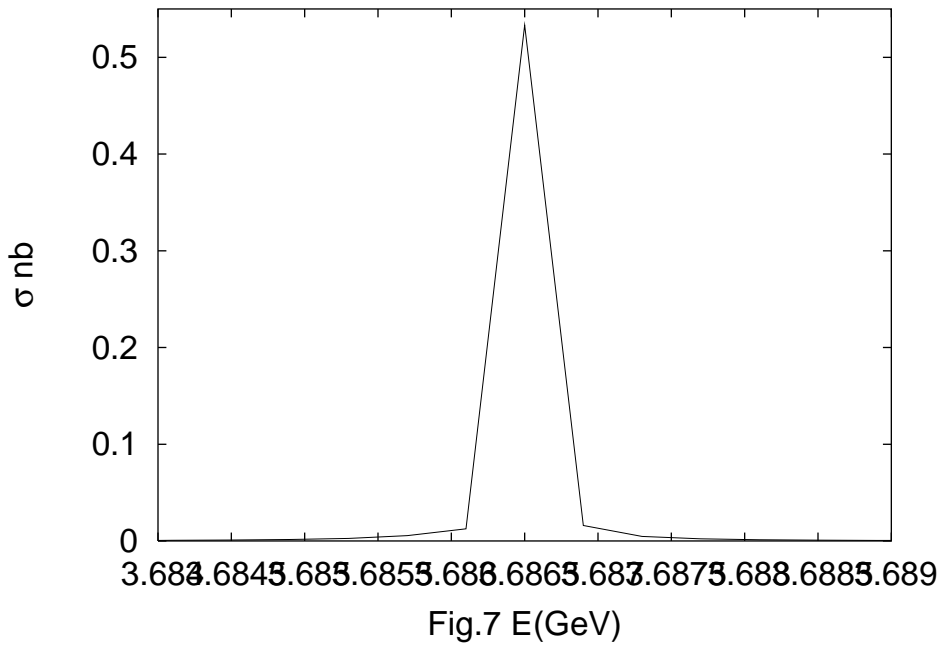


FIG. 7.



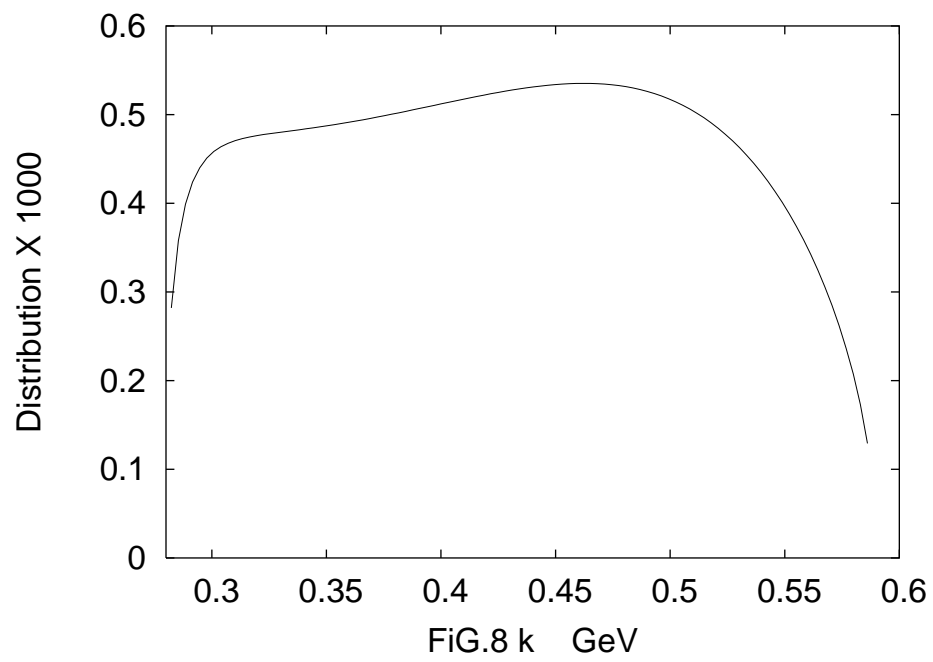


FIG. 8.

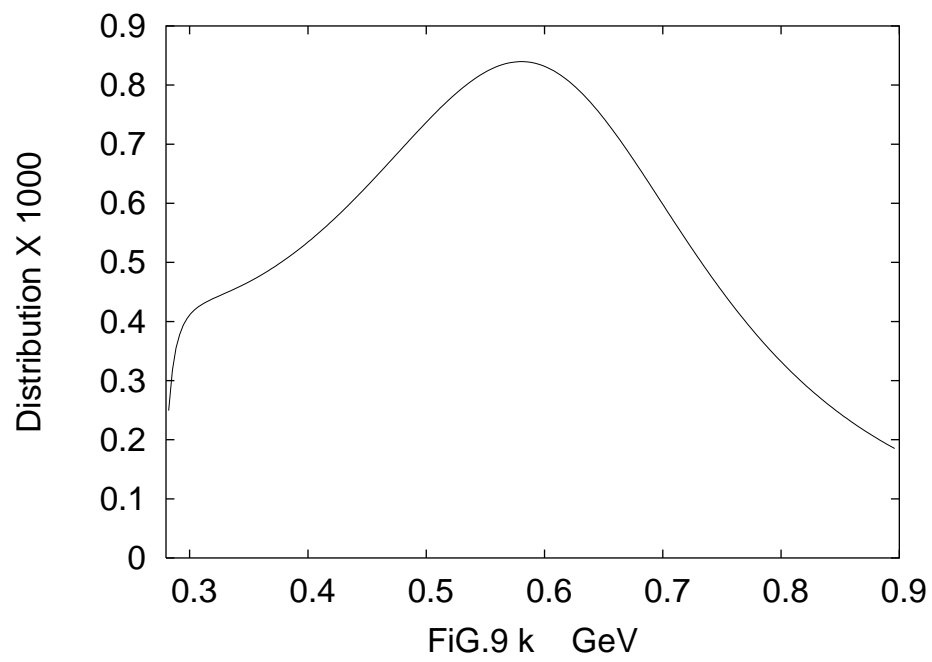


FIG. 9.